GLACIAL CYCLES: ROLE OF ECCENTRICITY IN ICE LINE MOVEMENT

MATH CLIMATE SEMINAR
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MILANKOVITCH CYCLES

Over the past one million years, glacial/interglacial cycles have occurred with periodicity of about 100,000 years.

Variations in the Earth's orbital parameters (obliquity, eccentricity, and precession) pace the glacial cycles.

COUPLED TEMPERATURE-ICE LINE MODEL

Time-dependent Energy Balance Model by Budyko:

$$R\frac{\partial T}{\partial t}(y,t) = Qs(y)\big(1-\alpha(y)\big) - (A+BT) - C(T-\bar{T}). \quad (1)$$

$Qs(y)\big(1-\alpha(y)\big)$ = **absorbed** term

$(A+BT)$ = **emitted** term.

$C(T-\bar{T})$ = **transport** term.

The albedo function is defined as:

$$\alpha_\eta(y) = \begin{cases} \alpha_1, & \text{if } y < \eta \\ \alpha_2, & \text{if } y > \eta, \end{cases}$$

where $\alpha_1 < \alpha_2$.

COUPLED TEMPERATURE-ICE LINE MODEL

Couple Budyko's equation with Widiasih's ODE in [4] for the evolution of the ice line, η:

$$\frac{d\eta}{dt} = \rho(T(\eta, t) - T_c). \qquad (2)$$

T_c is a critical temperature above which ice melts and below which ice forms.

QUADRATIC APPROXIMATION

The above infinite dimensional system ((1) and (2)) is approximated by the system of ODEs as done by McGehee and Widiaish in [1]:

$$\begin{cases} \dot{w} = -\tau\big(w - F(\eta)\big) \\ \dot{\eta} = \rho\big(w - G(\eta)\big) \end{cases} \qquad (3)$$

Where w, is a translate of the global average temperature. $F(\eta)$ (cubic polynomial) and $G(\eta)$(quadratic polynomial) are given below:

$$F(\eta) = \frac{1}{B}\left(Q(1-\alpha_0) - A + CL(\alpha_2 - \alpha_1)\left(\eta - \frac{1}{2} + s_2 P_2(\eta)\right)\right),$$

$$G(\eta) = -Ls_2(1-\alpha_0)p_2(\eta) + T_c.$$

QUADRATIC APPROXIMATION

In [5], McGehee and Lehman proved that:

$$Q = Q(e) = \frac{Q_0}{\sqrt{1-e^2}},$$

and in [2], McGehee and Widiasih showed that:

$$s_2(\beta) = \frac{5}{16}(-2 + 3\sin^2\beta).$$

From [1]:

$$L = \frac{Q}{B + C},$$

$$P_2(\eta) = \frac{1}{2}(\eta^3 - \eta),$$

$$p_2(\eta) = \frac{1}{2}(3\eta^2 - 1).$$

QUADRATIC APPROXIMATION

Parameter	Value	Units
Q_0	343	$\mathrm{W\,m^{-2}}$
A	202	$\mathrm{W\,m^{-2}}$
B	1.9	$\mathrm{W\,m^{-2}K^{-1}}$
C	3.04	$\mathrm{W\,m^{-2}K^{-1}}$
α_1	0.32	dimensionless
α_2	0.62	dimensionless
$\alpha_0 = \frac{\alpha_1 + \alpha_2}{2}$	0.47	dimensionless
T_c	-5.5 and -10	°C

EQUILIBRIUM SOLUTIONS

$$\begin{cases} \dot{w} = -\tau\big(w - F(\eta)\big) \\ \dot{\eta} = \rho\big(w - G(\eta)\big) \end{cases}$$

In order to find equilibrium solutions of the system above, we set the derivatives to 0, and we get

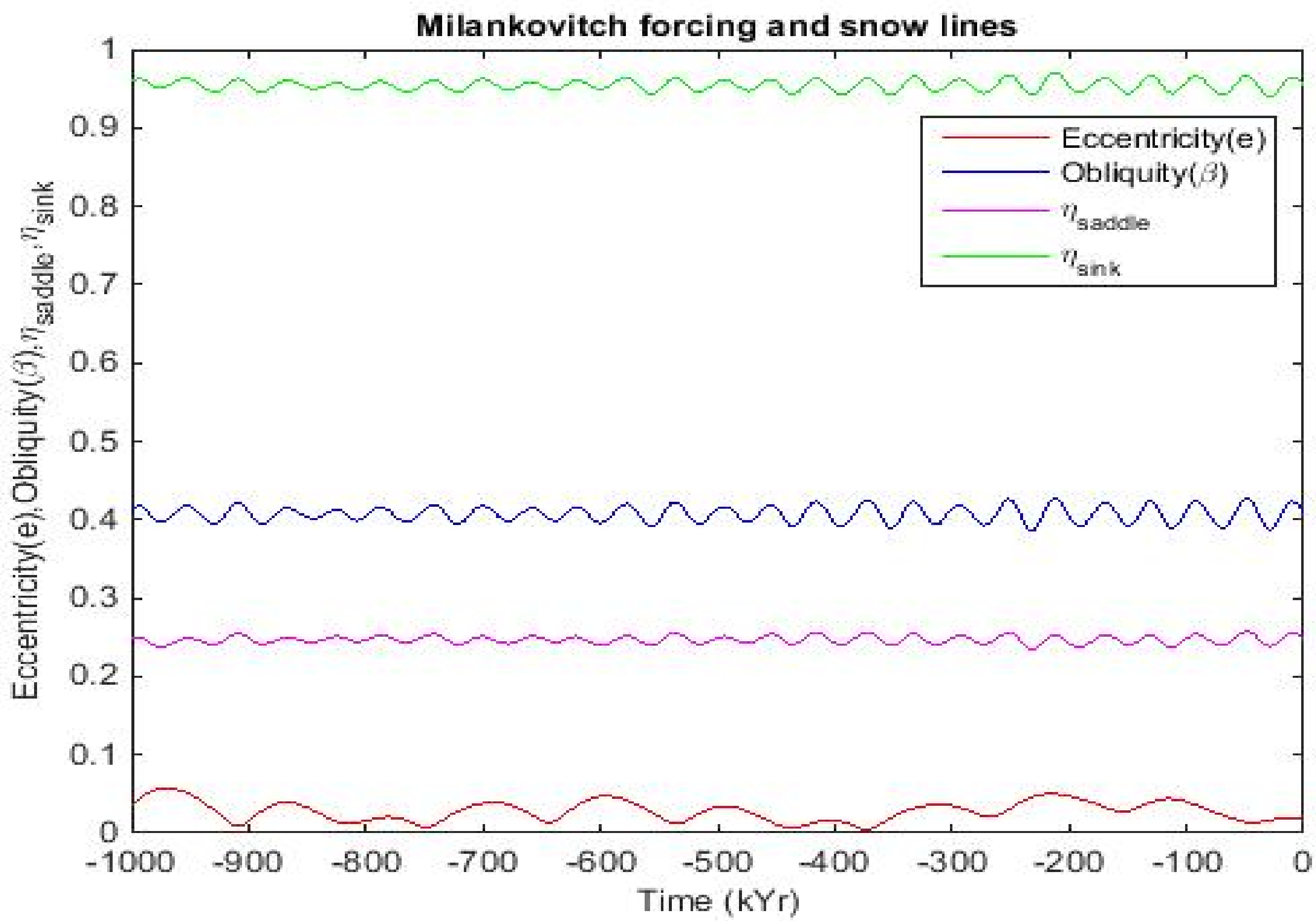
$$w = F(\eta) = G(\eta).$$

Now, we may solve for η in the equation

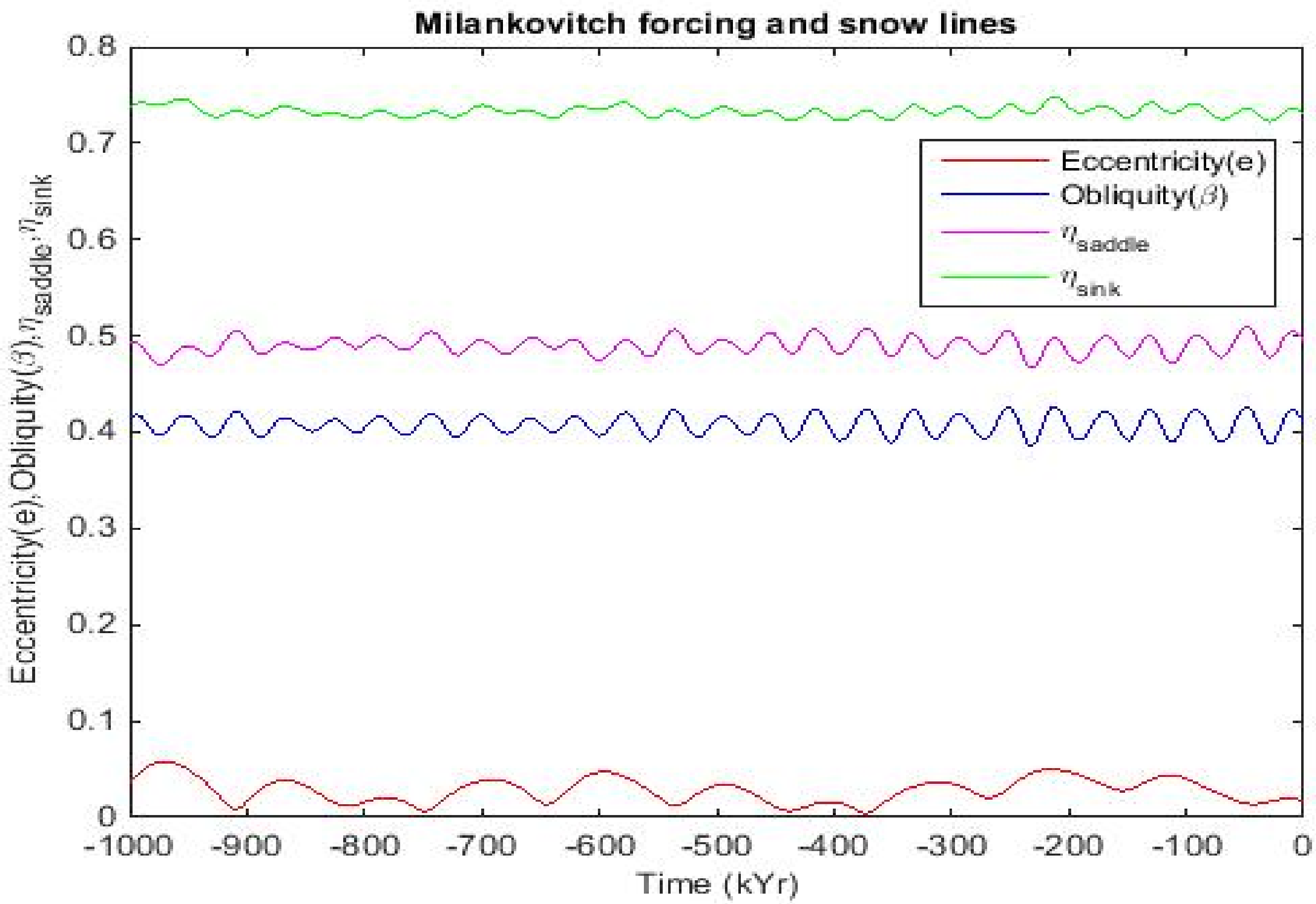
$$F(\eta) - G(\eta) = 0.$$

Three roots are found, and one is discarded since it doesn't belong to the range $[0,1]$.

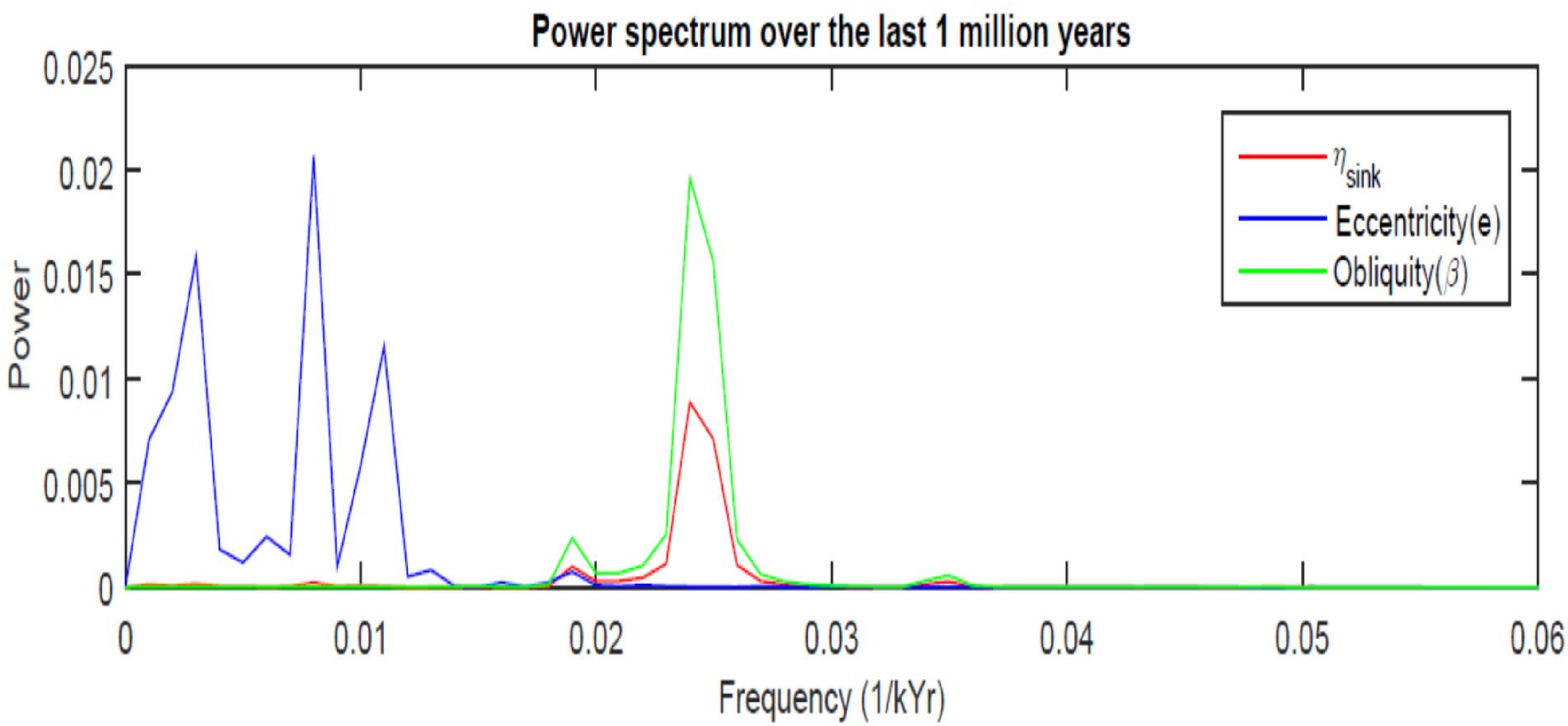
MILANKOVITCH FORCING AND ICE LINES AT $T = -10°C$



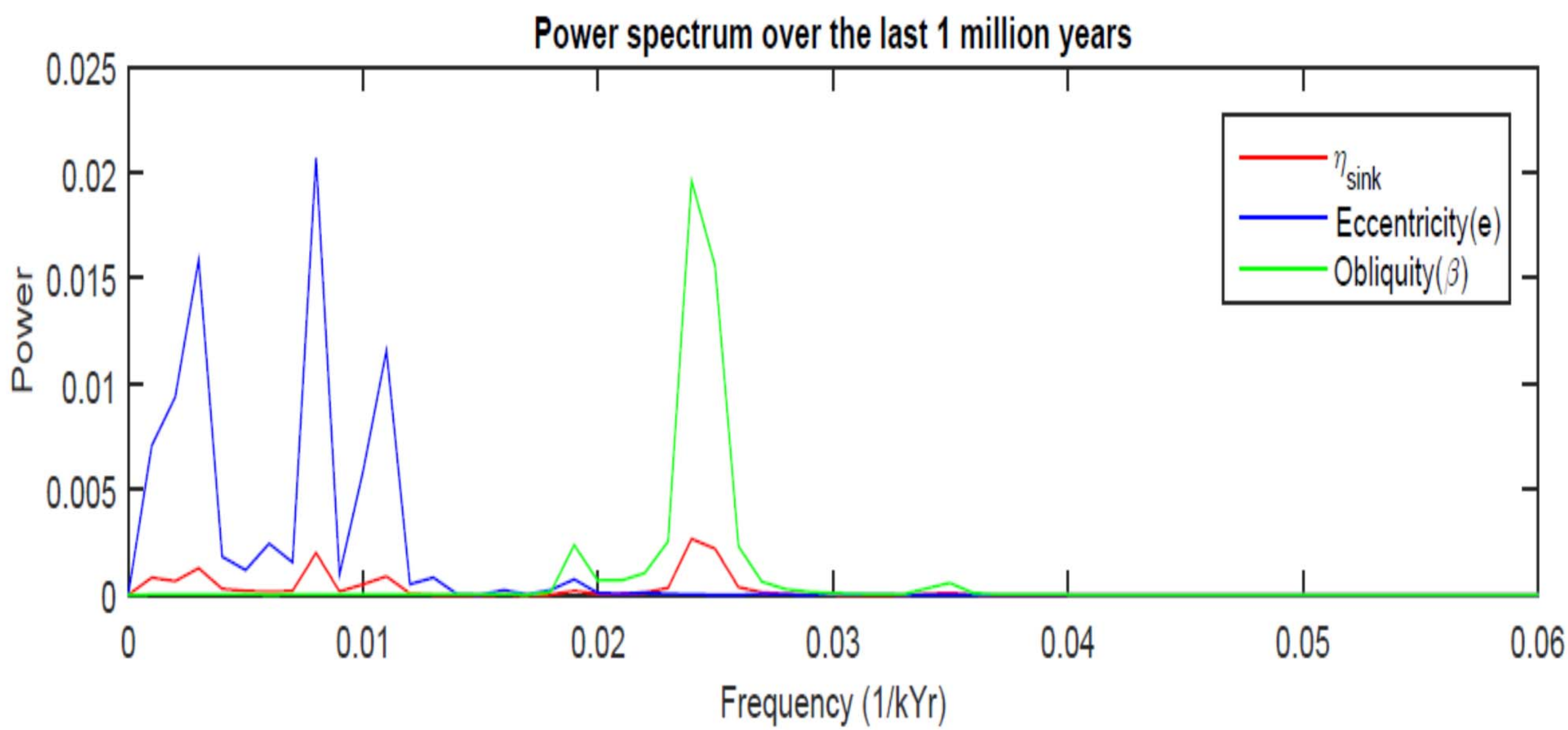
MILANKOVITCH FORCING AND ICE LINES AT $T = -5.5°C$



POWER SPECTRUM
$T_c = -10°C$ (SINK - SMALLER ICE CAP)



POWER SPECTRUM
$T_c = -5.5°C$ (SINK - LARGER ICE CAP)



REVISIT MODEL

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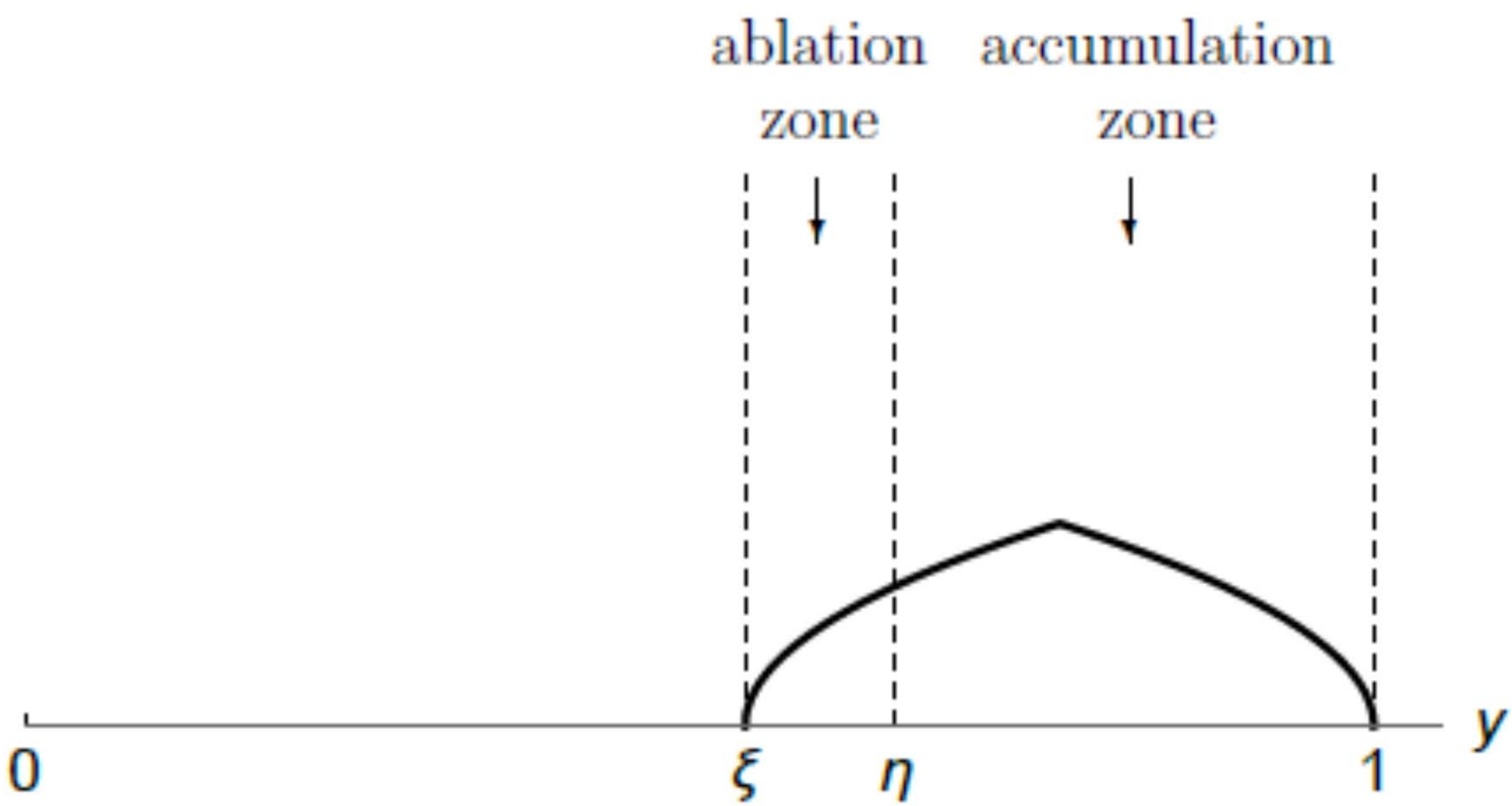
ADDITION OF SNOW LINE



ADDITION OF SNOW LINE

$B = \{(w, \eta, \xi): w \in \mathbb{R}, \eta \in [0,1], \xi \in [0,1]\}$,
$b_0 < b < b_1 =$ ablation rates,
$a =$accumulation rate.
When $b(\eta - \xi) - a(1 - \eta) < 0$, set $T_c = -5.5$ °C and

$$\begin{cases} \dot{w} = -\tau(w - F(\eta)) \\ \dot{\eta} = \rho(w - G_-(\eta)) \\ \dot{\xi} = \epsilon\big(b_0(\eta - \xi) - a(1 - \eta)\big). \end{cases} \quad (4)$$

When $b(\eta - \xi) - a(1 - \eta) > 0$, set $T_c = -10$ °C and

$$\begin{cases} \dot{w} = -\tau(w - F(\eta)) \\ \dot{\eta} = \rho(w - G_+(\eta)) \\ \dot{\xi} = \epsilon\big(b_1(\eta - \xi) - a(1 - \eta)\big). \end{cases} \quad (5)$$

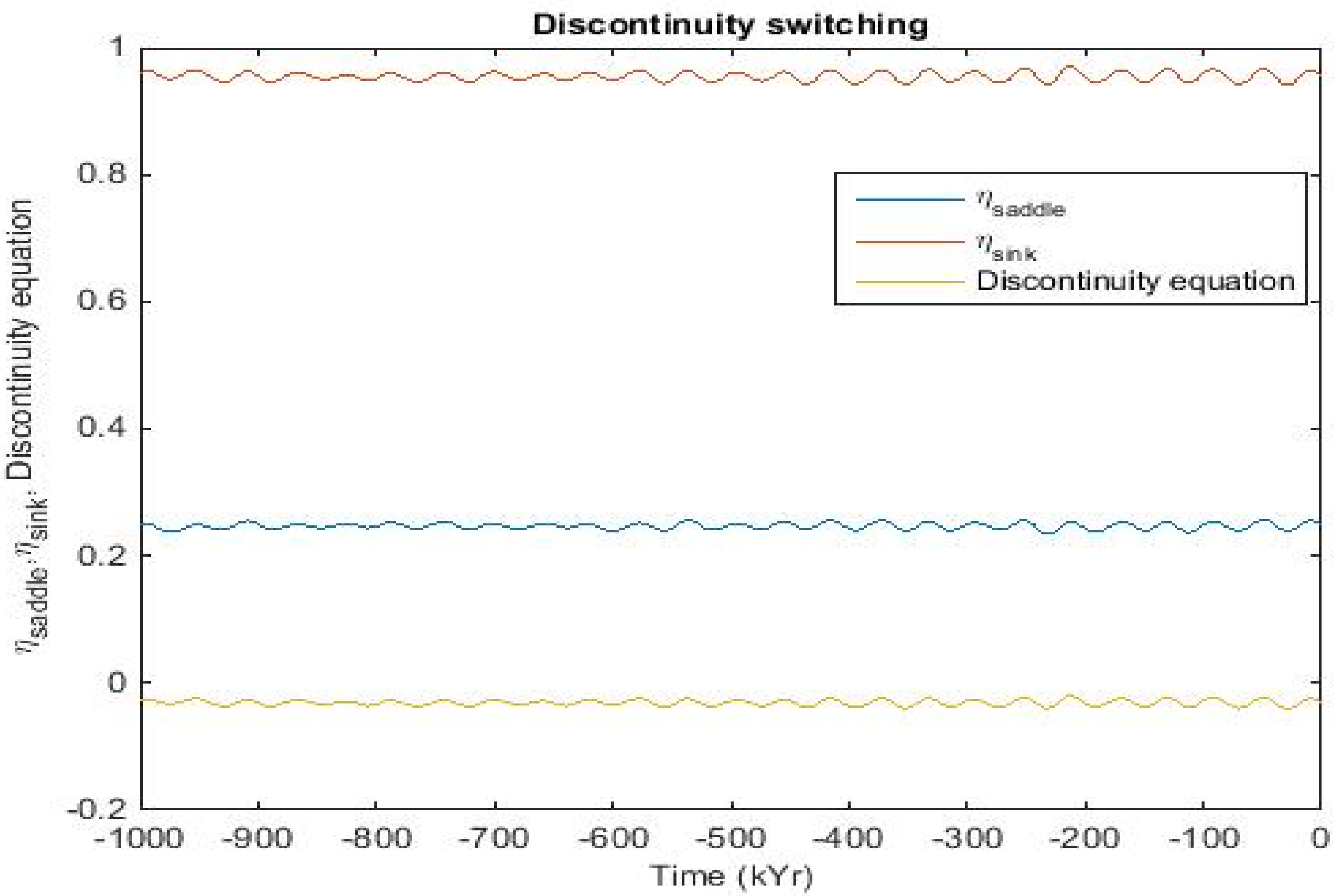
ADDITION OF SNOW LINE

We thus arrive at a 3-dimensional system having a plane of discontinuity [3]:

$$\Sigma = \{(w, \eta, \xi) : b(\eta - \xi) - a(1 - \eta) = 0\}$$
$$= \left\{(w, \eta, \xi) : \xi = \left(1 + \frac{a}{b}\right)\eta - \frac{a}{b}\right\}.$$

Constants	Value
a	1.05
b_0	1.5
b	1.75
b_1	5

$b = 1.75, a = 1.05$
$D = b(\eta - \xi) - a(1 - \eta)$
$T_c = -10\ ℃$



RECALL

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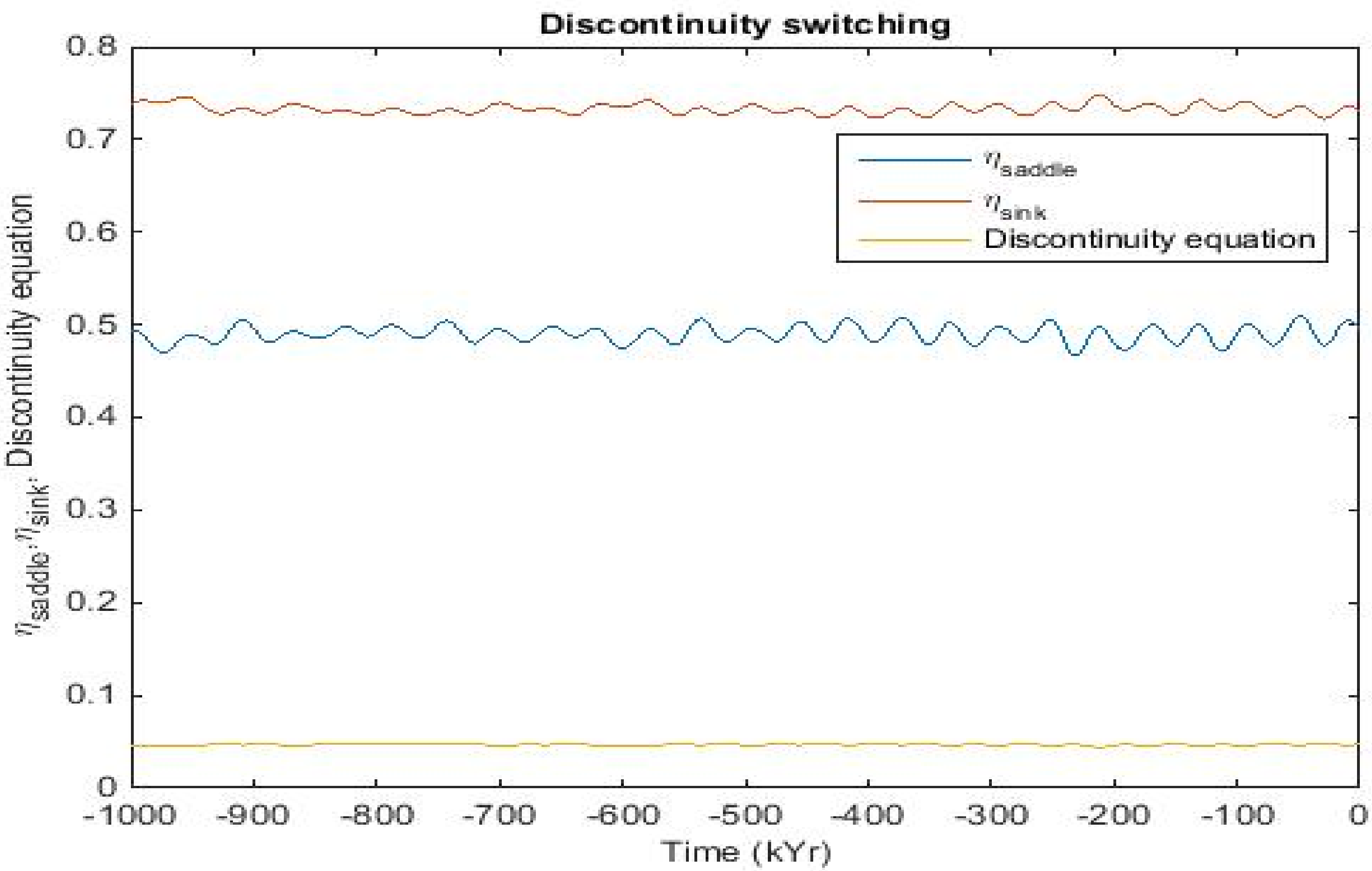

RECALL

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$$\begin{cases} \dot{w} = -\tau(w - F(\eta)) \\ \dot{\eta} = \rho(w - G_+(\eta)) \\ \dot{\xi} = \epsilon\big(b_1(\eta - \xi) - a(1 - \eta)\big). \end{cases}$$

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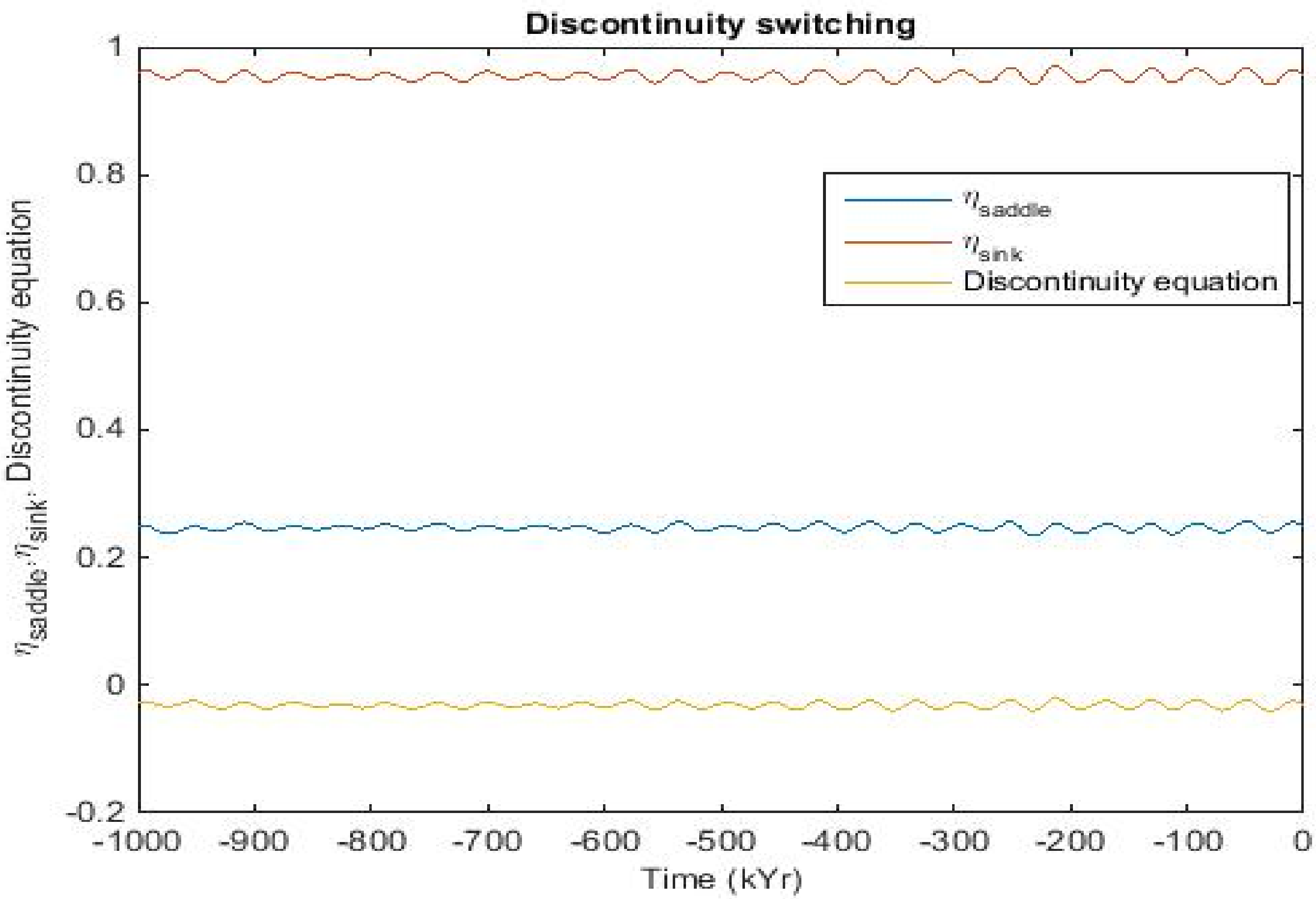

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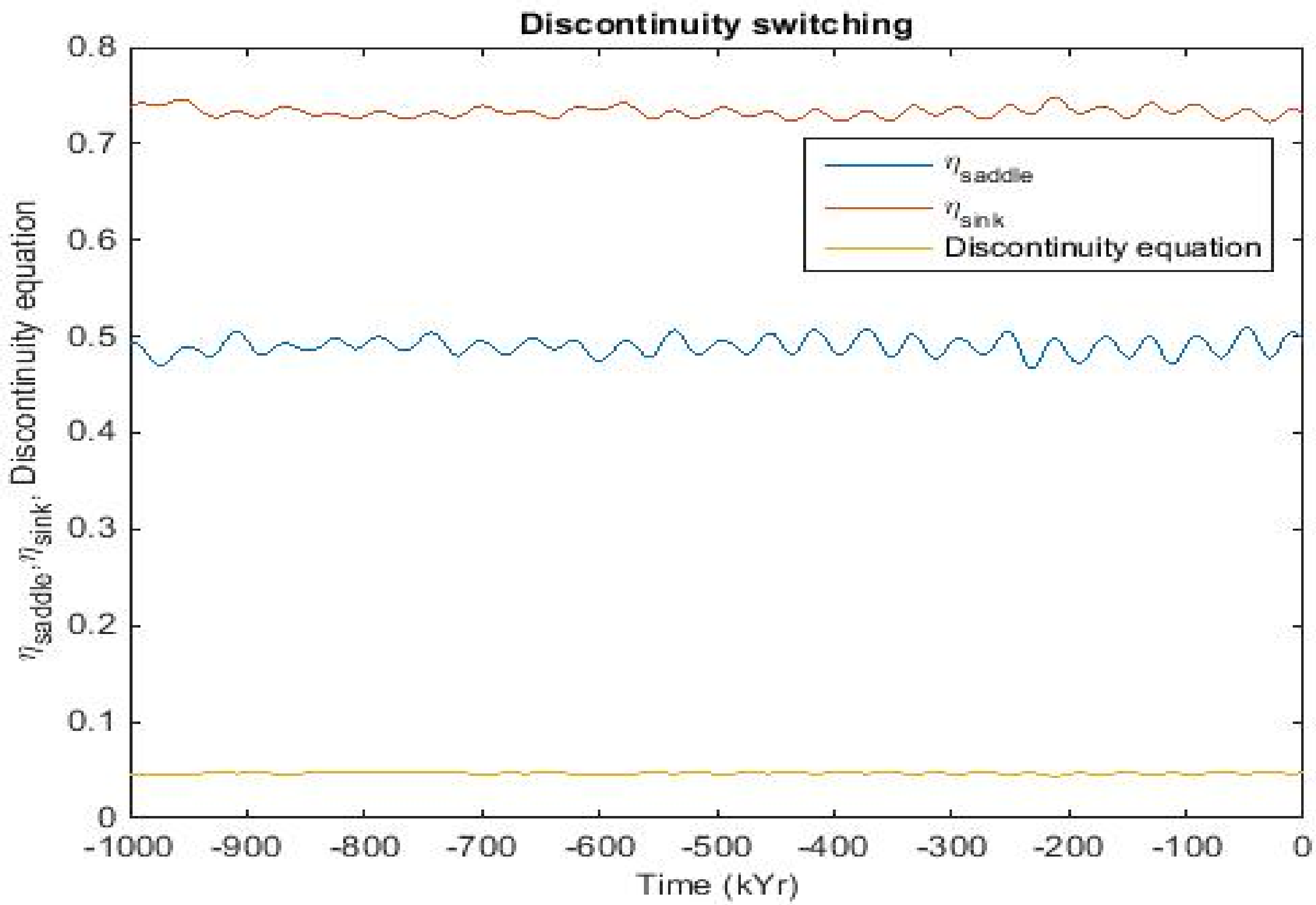

RECALL

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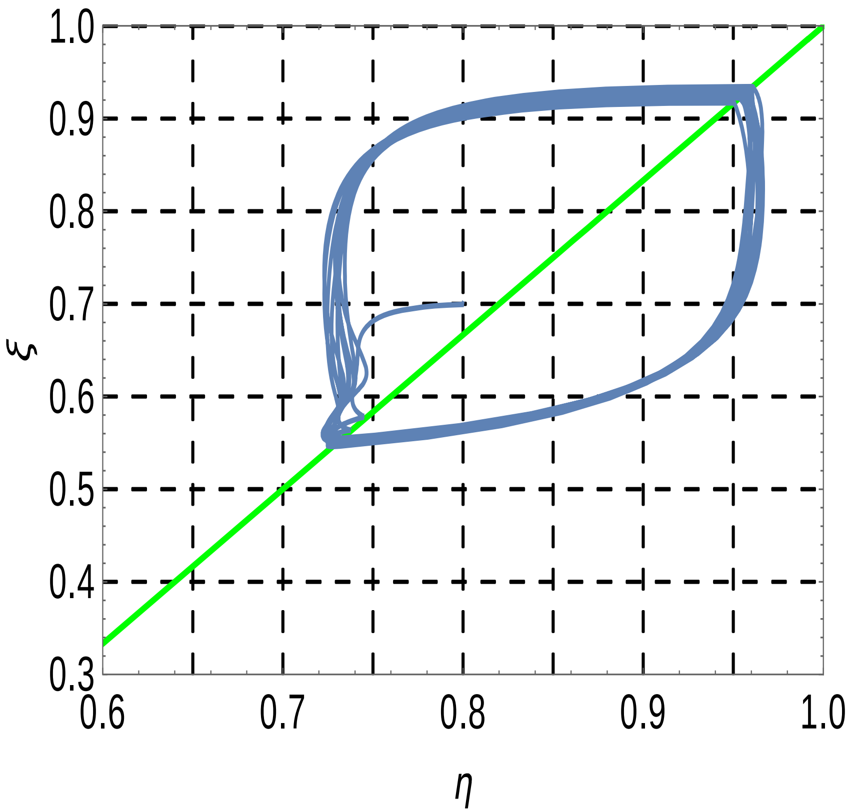
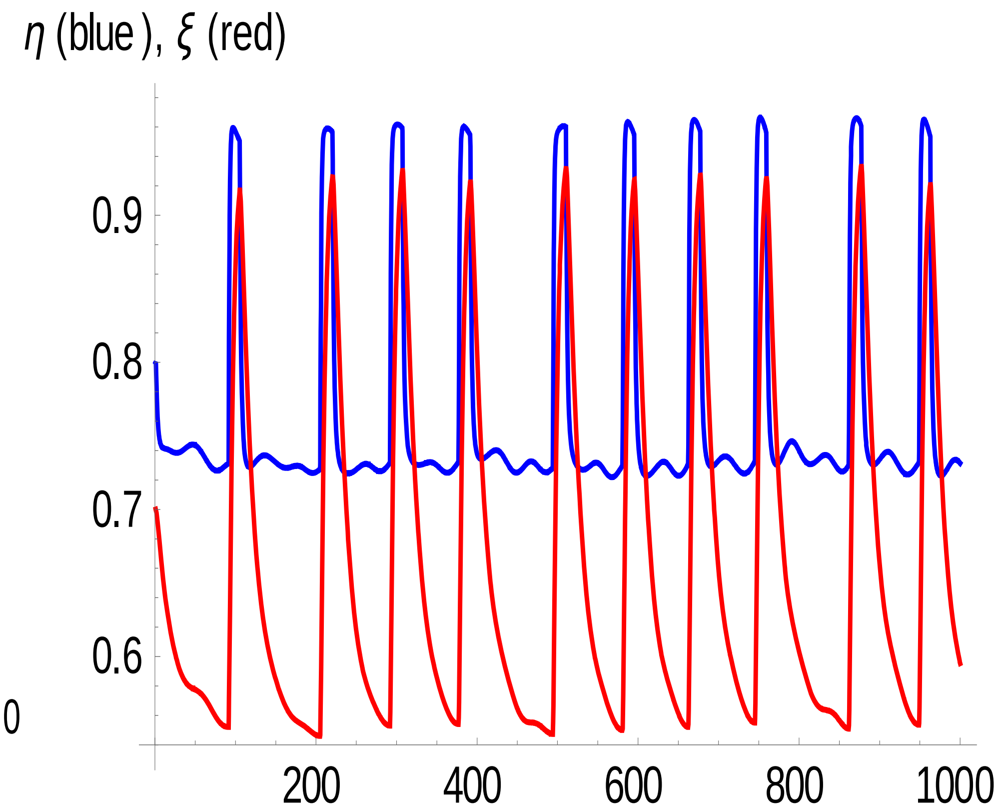
FULL MILANKOVITCH FORCING

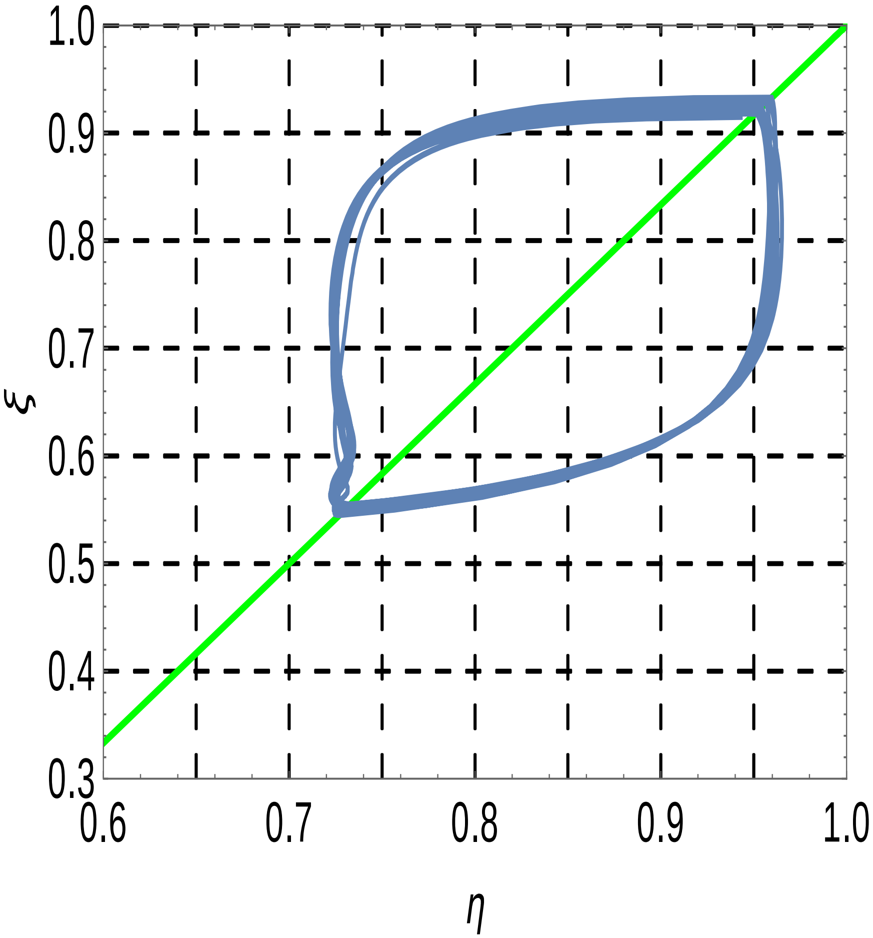
$$b = 1.5, b_1 = 5,\ a = 1,\ \rho = \epsilon = 4 \times 10^{-2}$$

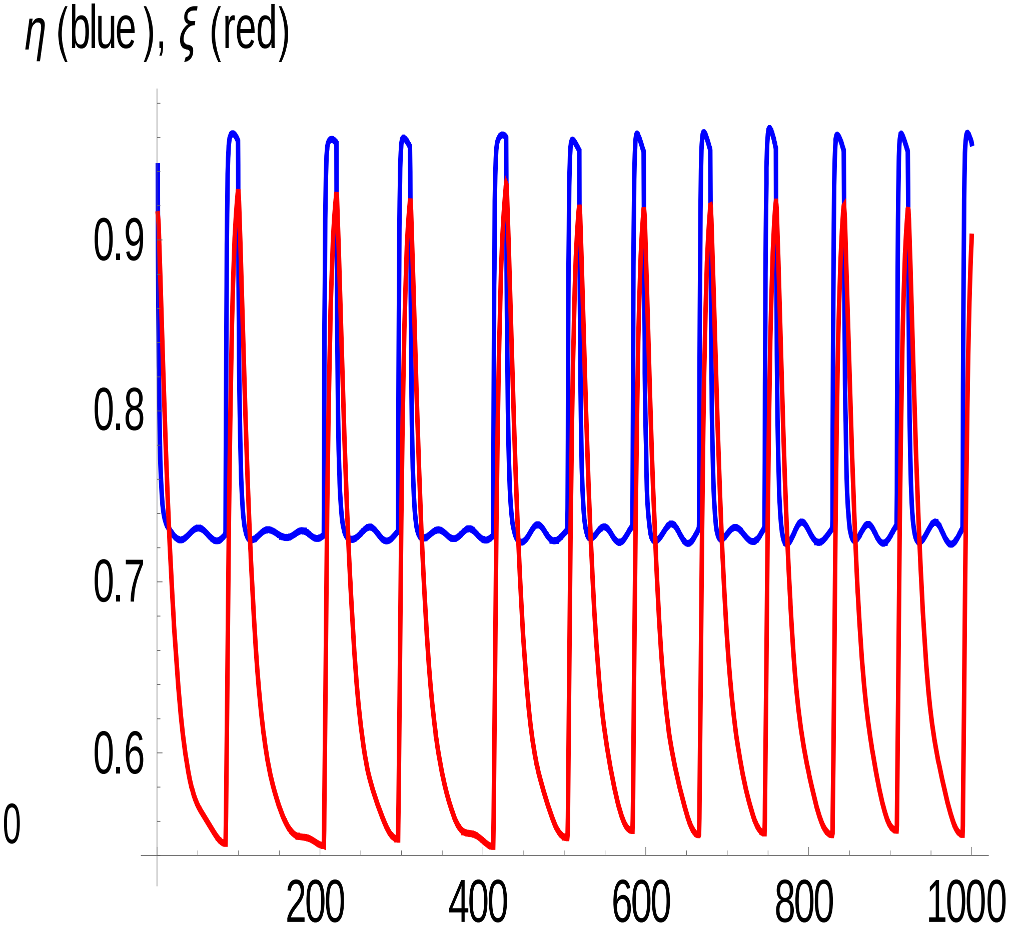




OBLIQUITY ONLY FORCING

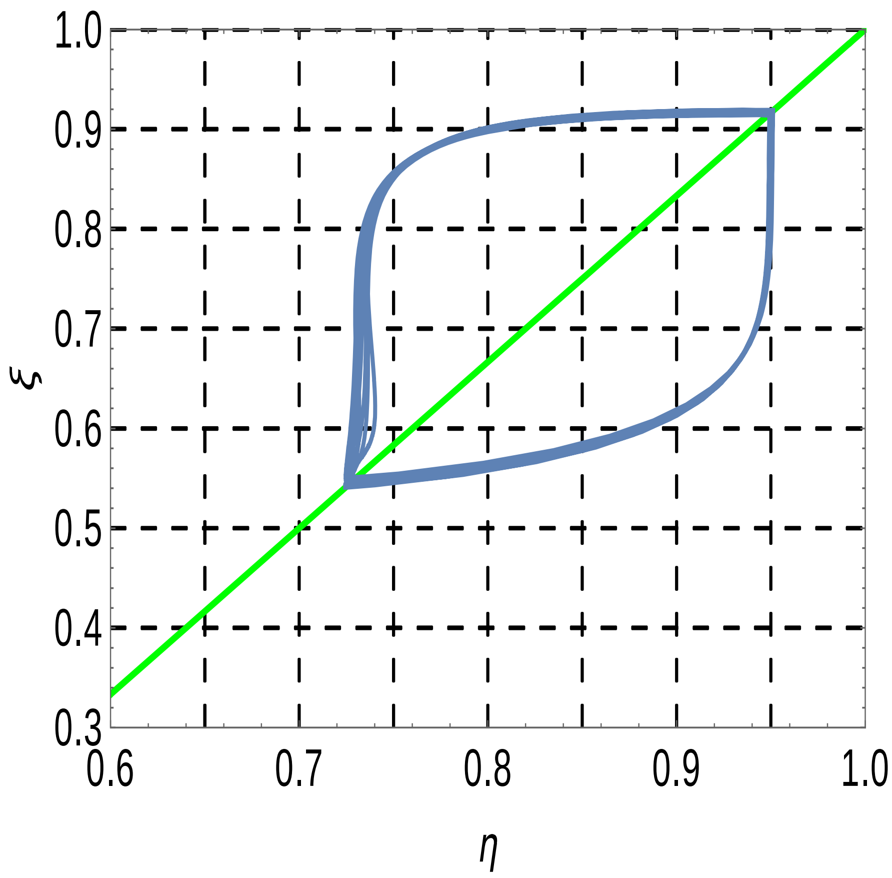
$b = 1.5, b_1 = 5,\ a = 1,\ \rho = \epsilon = 4 \times 10^{-2}$

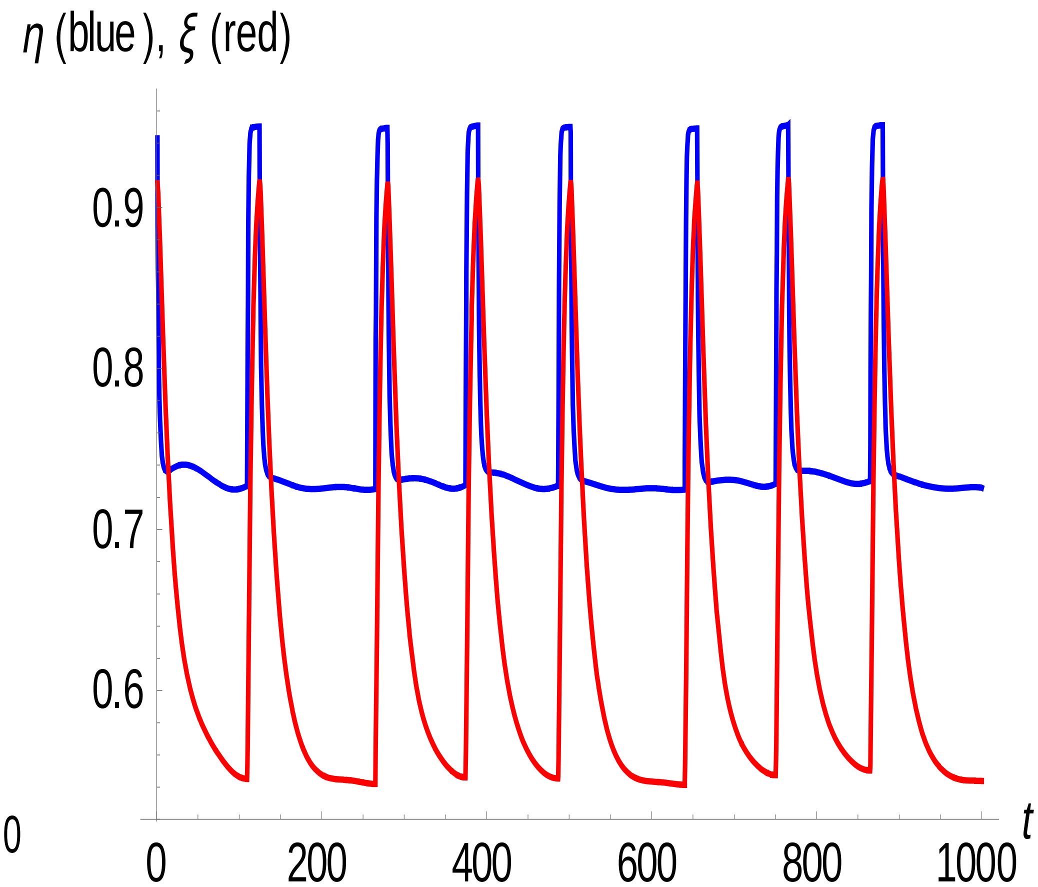




ECCENTRICITY ONLY FORCING

$b = 1.5, b_1 = 5,\ a = 1,\ \rho = \epsilon = 4 \times 10^{-2}$





DISCUSSION AND FURTHER WORK

Use a more complex model backed by intensive numerical simulations to verify the role eccentricity plays.

Why is η_{saddle} in the $T = -5.5°\text{C}$ case perfectly in-sync with the obliquity variation whereas η_{sink} is forced by eccentricity?

Long shot: Relate to the Mid-Pliestocene Transition – shift from 41 kYr (obliquity forced) cycles to 100 kYr cycles

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[2] R. McGehee & E. Widiasih, A simplification of Budyko's ice-albedo feedback model, March 2012.

[3] J. Hahn, R. McGehee, J. Walsh, and E. Widiasih. "Conceptual Glacial Cycle Model", July 2015 (currently being reviewed for publication).

[4] Widiasih, Esther R. "Dynamics of the Budyko energy balance model." *SIAM Journal on Applied Dynamical Systems* 12.4 (2013): 2068-2092.

[5] McGehee, Richard, and Clarence Lehman. "A Paleoclimate Model of Ice-Albedo Feedback Forced by Variations in Earth's Orbit." *SIAM Journal on Applied Dynamical Systems* 11.2 (2012): 684-707.